

REVIEW
of the official reviewer for dissertation work
BAYAN BEKBOLAT on the theme «Dunkl analysis and application to inverse source problems»
presented for the degree of Doctor of Philosophy (PhD) in the specialty «6D060100 – Mathematics».

№	Criteria	Eligibility (one of the options must be checked)	Justification of the position of the official reviewer
1.	The topic of the thesis (as of the date of its approval) corresponds to the directions of development of science and/or state programs	1.1 Compliance with priority areas of science development or government programs: 1) The thesis was completed within the framework of a project or target program financed from the state budget (indicate the name and number of the project or program) 2) The thesis was completed within the framework of another state program (indicate the name of the program) 3) The dissertation corresponds to the priority direction of the development of science, approved by the Higher Scientific and Technical Commission under the Government of the Republic of Kazakhstan (indicate the direction)	Case 1: The thesis was completed within the framework of a project financed from the state budget of Republic of Kazakhstan AP14972634 «Inverse source problems for pseudo-parabolic equations associated with Dunkl and Jacobi operators» («Zhhas Galym», 2022 – 2024).
2.	Importance for science	The work makes/does not make a significant contribution to science, and its importance is well disclosed/not disclosed	The work makes a significant contribution to the theory of pseudo-differential operators generated by the Dunkl operators and to the theory of inverse source problems for the parabolic and pseudo-parabolic equations. The importance is well-disclosed.
3.	The principle of independence	Self-reliance level: 1) <u>High</u> ; 2) <u>Medium</u> ; 3) <u>Low</u> ;	I have noticed that all of the results are published with several co-authors and no paper has a unique author. I have estimated that level as a medium for that reason.

		4) No independence	
4.	The principle of inner unity	<p>4.1 Justification of the relevance of the thesis: 1) Justified; 2) Partially justified; 3) Not justified.</p> <p>4.2 The content of the thesis reflects the topic of the thesis: 1) Reflects; 2) Partially reflects; 3) Does not reflect</p> <p>4.3. The purpose and objectives correspond to the topic of the thesis: 1) correspond; 2) partially correspond; 3) do not correspond</p> <p>4.4 All sections and provisions of the thesis are logically interconnected: 1) completely interconnected; 2) the interconnection is partial; 3) there is no interconnection</p> <p>4.5 The new solutions (principles, methods) proposed by the author are reasoned and evaluated in comparison with the known solutions: 1) there is a critical analysis; 2) partial analysis; 3) the analysis does not represent one's own opinions, but quotes from other authors</p>	<p>The thesis is logically subsequent.</p> <p>The content reflects the topic.</p> <p>The purpose and objectives correspond to the topic of the thesis</p> <p>All sections and provisions of the thesis are completely interconnected</p> <p>There is a critical analysis, and the new results proposed by the author are reasoned and evaluated compared to the known solutions.</p>
5.	Scientific novelty principle	<p>5.1 Are the scientific results and provisions new? 1) completely new; 2) partially new (25-75% are new); 3) not new (less than 25% are new)</p> <p>5.2 Are the dissertation findings new? 1) completely new; 2) partially new (25-75% are new); 3) not new (less than 25% are new)</p> <p>5.3 Technical, technological, economic or management decisions are new and reasonable:</p>	<p>The main results are completely new. Other results that are included in the text are given with appropriate references.</p> <p>The main findings are new; the author has made the generalization of Dunkel analysis and pseudo-differential operators.</p> <p>There are no technological, economic, or management decisions,</p>

		1) completely new; 2) partially new (25-75% are new); 3) not new (less than 25% are new)	this criterion is inapplicable.
6.	The validity of the main findings	All main conclusions are/are not based on scientifically significant evidence or well-grounded (for qualitative research and areas of training in the arts and humanities)	All main conclusions are well-grounded
7.	The main provisions for the defense	<p>It is necessary to answer the following questions for each provision separately:</p> <p>7.1 Is the provision proven? 1) proven; 2) rather proven; 3) rather not proven; 4) not proven</p> <p>7.2 Is it trivial? 1) yes; 2) no</p> <p>7.3 Is it new? 1) yes; 2) no</p> <p>7.4 Application level: 1) narrow; 2) <u>medium</u>; 3) wide</p> <p>7.5 Is it proven in the article? 1) yes; 2) no</p>	<p>The main provisions are:</p> <p>1) proven, 2) not trivial, 3) new.</p> <p>4) The application level is quite narrow; it is restricted by very special areas of functional analysis and inverse problems</p> <p>5) The results are published and proven in articles</p>

8.	The principle of reliability Reliability of sources and information provided	8.1 Choice of methodology - is justified or the methodology is described in sufficient detail 1) <u>yes</u> ; 2) no	The methodology is quite standard and based on the classical theories of Bessel functions, Fourier transform, Schwartz spaces, and generalized functions (distributions)
		8.2 The results of the thesis were obtained using modern methods of scientific research and methods of processing and interpreting data using computer technologies: 1) <u>yes</u> ; 2) no	The main content of the thesis is theoretical; there is no need to apply high computing technologies. However, the author has applied some simple computer techniques and software to show several numeric examples
		8.3 Theoretical conclusions, models, identified relationships and patterns have been proven and confirmed by experimental research (for areas of training in pedagogical sciences, the results have been proven on the basis of a pedagogical experiment): 1) <u>yes</u> ; 2) no	Not applicable
		8.4 Important statements are confirmed / partially confirmed / not confirmed by references to current and reliable scientific literature 8.5 Used literature sources are sufficient/not sufficient for a literature review	Important statements are confirmed by references to current and reliable scientific literature The used literature sources are sufficient for a literature review
9	Practical value principle	9.1 The thesis has theoretical value: 1) <u>yes</u> ; 2) no	The work has rather theoretical value, and there is no evidence for its immediate practical application
		9.2 The thesis is of practical importance and there is a high probability of applying the results obtained in practice: 1) <u>yes</u> ; 2) <u>no</u>	Because of the theoretical nature of the work and its specific topic, I do not see current practical application, so there is no high probability of applying the results in practice.
		9.3 Are the practice suggestions new? 1) completely new; 2) partially new (25-75% are new);	Not applicable to that work

		3) not new (less than 25% are new)	
10.	The quality of writing and design	Academic writing quality: 1) high; 2) <u>average</u> ; 3) below average; 4) low.	In the introduction part, the main drawback is that the author first uses the new notation and explains and defines it several pages later: This produces many difficulties for readers not currently working on that topic.

I indicate the following solution: awarding the Doctor of Philosophy (PhD) to Bayan Bekbolat.

Copies of the official reviewers' reviews are handed over to the doctoral student no later than 5 (five) working days before the defense of the thesis.

Official Reviewer:

Department of Computations and Data Science

Astana IT University,

Professor _____

(place of work, academic title)


(signature)

____Balgaisha Mukanova____
(FULL NAME)

14 February 2024

